SAMPLE PROBLEMS FOR THE QUIZ

Problem 1. Let F be a field and let $F(t)$ be the field of rational functions over F. Let $s = f(t) \in F[t]$ be a polynomial of positive degree.

(a) Show that $F(t) \supset F(s)$ is an algebraic extension and compute the degree $[F(t) : F(s)]$.

(b) Show that $F(s) \supset F$ is transcendental.

Problem 2. Prove that the polynomial $x^4+8x^3+19x^2+12x+6$ is irreducible in $\mathbb{Q}[x]$.

Problem 3. Let $\alpha = \sqrt[3]{11}$ be a third root of 11, and let $K = \mathbb{Q}(\alpha)$.

(a) Compute $[K : \mathbb{Q}]$.

(b) Is $[K : \mathbb{Q}]$ normal?

(c) Compute the Galois group $Gal(K/\mathbb{Q})$.

Problem 4. Let $L \supset K$ be a field extension such that $[L : K]$ is prime. Prove that the extension is simple.

Problem 5. For each of the following claims, state whether it is true or false. Prove or give a counterexample.

(a) Any field K admits a nontrivial finite algebraic extension.

(b) Any simple extension is algebraic.